

Study Guide

Open Sentences

Mathematical statements with one or more variables are called **open sentences**. Open sentences are **solved** by finding a replacement for the variable that results in a true sentence. The replacement is called a **solution**.

Example 1: Replace a in $3a + 12 = 39$ with the value 9.

$$\begin{aligned} 3a + 12 &= 39 \\ 3(9) + 12 &= 39 \\ 27 + 12 &= 39 \\ 39 &= 39 \quad \text{true} \end{aligned}$$

Since $a = 9$ makes the sentence $3a + 12 = 39$ true, 9 is a solution.

A set of numbers from which replacements for a variable may be chosen is called a **replacement set**. The set of all replacements for the variable in an open sentence that results in a true sentence is called the **solution set** for the sentence.

A sentence that contains an equals sign, $=$, is called an **equation** and sometimes may be solved by simply applying the order of operations. A sentence having the symbols $<$ or $>$ is called an **inequality**.

Example 2: Solve $\frac{2(3 + 1)}{3(2 + 1)} = b$.

$$\begin{aligned} \frac{2(3 + 1)}{3(2 + 1)} &= b \\ \frac{2(4)}{3(3)} &= b \\ \frac{8}{9} &= b \end{aligned}$$

State whether each equation is true or false for the value of the variable given.

1. $y + \frac{1}{2} = \frac{1}{4} + \frac{1}{2}, y = \frac{7}{2}$

2. $x^4 = 2^8, x = 4$

3. $a^2 + a^3 + 2 < 10, a = 2$

4. $\frac{2^3 - 2d}{3^2 - 1} \leq 2, d = 2$

Find the solution set for each inequality if the replacement sets are $x = \{\frac{1}{2}, 2, 3, \frac{1}{4}\}$ and $y = \{2, 4, 6, 8\}$.

5. $x + 4 < 6$

6. $3y \geq 18$

7. $\frac{y}{3} > 1$

Solve each equation.

8. $s = \frac{15 - 6}{27 - 24}$

9. $w = 6^2 - 3^3$

10. $c = 3\frac{1}{2} + 2\frac{1}{4}$