Worksheet

Logarithmic Functions

Introduction

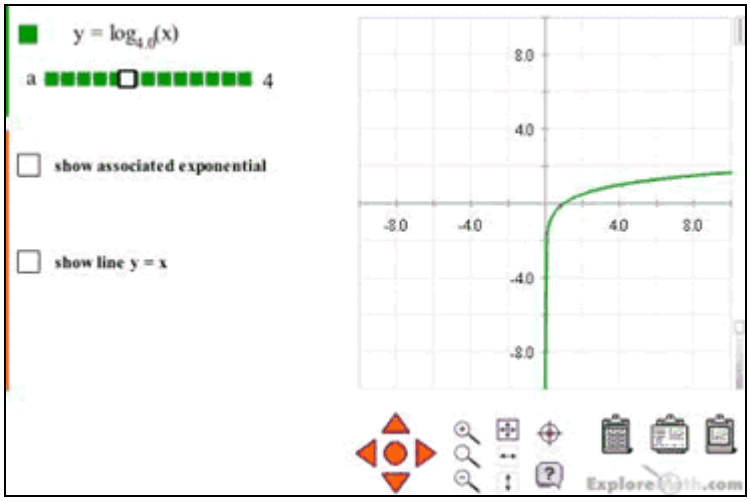
Logarithmic functions are the inverses of exponential functions. Logarithms are found in applications such as compound interest, earthquake magnitudes, and the pH of solutions. In this activity we will look at logarithmic functions in the form $\mathbf{y = \log_a x}$.

Let's explore logarithmic functions. Use your web browser to go to the "Logarithms" activity:

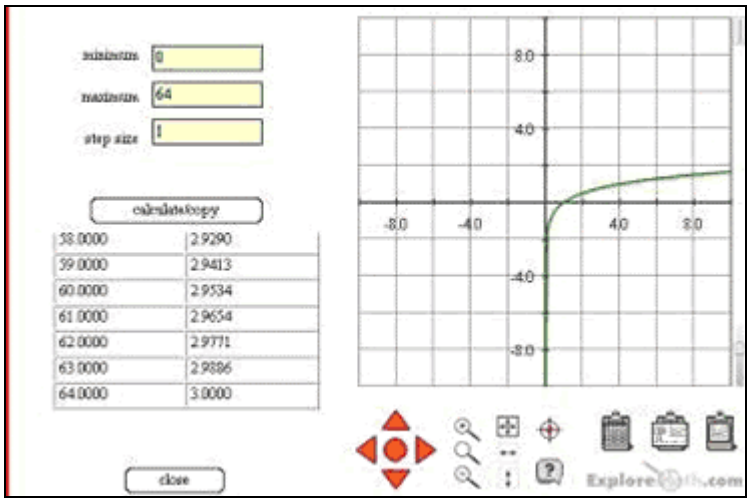
http://www.exploremath.com/activities/Activity_page.cfm?ActivityID=7

Below are several questions designed to get you thinking about the activity. Answer them on a separate sheet of paper.

Defining $y = \log_a x$
Grab the '**a**' slide bar and move it such that **a=4**. This can also be accomplished by typing a **4** to the right of the slide bar.



Now select the "Calculate Data Values" clipboard at the bottom of the screen. Then set the minimum to **0**, set the maximum to **64** and set the step to **1**.



Pick out the **x**, **y** pairs that have integer values. These should be:

x	1	4	16	64
y	0	1	2	3

Remember that the base, **a**, of this logarithm is **4**.

Question 1a. How does '**a**', '**y**', and '**x**' related to each other?

Question 1b. What conjecture can you make about the definition of $y = \log_a x$?

Question 1c. What is the value of **y** in the equation $y = \log_3 81$?

Restrictions of logarithmic functions

Grab the '**a**' slide bar and slide it to the left and right.

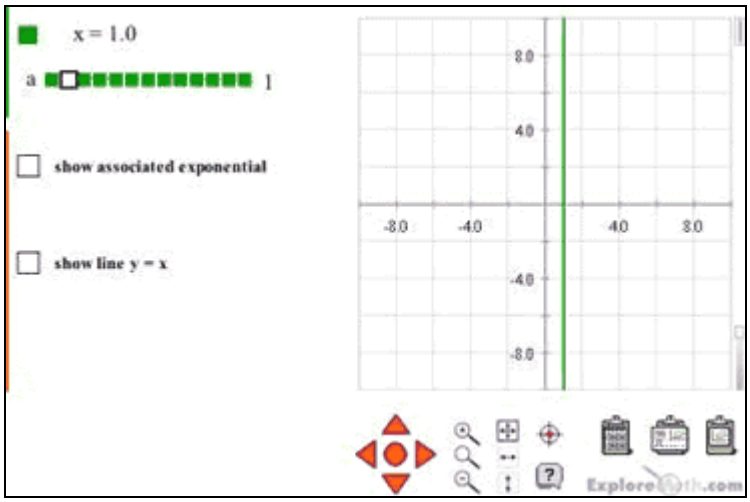
Question 2a. What happens to the graph when '**x**' approaches the y-axis?

Question 2b. Does the graph ever cross the y-axis?

Question 2c. Why do the graphs never enter the second or third quadrants? Remember that $a^y = x$.

Question 2d. Why is '**a**' always positive? Think about inverses of exponential functions.

Question 2e. Why can't **a** =**1** in a logarithmic function? Set **a** = **1** and observe the graph.



Question 2f. What point does the graphs of logarithmic functions pass through regardless of the base? Experiment with the 'a' slide bar to help find the answer.

Algebraic inverses

Type e^5 into your calculator. Now take the '**ln**' of the answer.

Question 3a. What do you notice about the final answer compared to the original expression?

Question 3b. What would be the solution of $\log_8(8^5)$? Why?

Question 3c. What would be the solution of $e^{(\ln 9)}$? Why?

Applications

Question 4a. If **$200** were invested in an account where the interest is compounded continuously at a rate of **5%**, the amount, **A**, in the account after **t** years could be evaluated using the equation: $A = 200e^{.05t}$. How long would it take the account to accumulate to **$500**?

The pH of a chemical is given by the formula: $PH = -\log_{10}[H^+]$, where $[H^+]$ is the concentration of hydrogen ions in moles per liter.

Question 4b. What is the hydrogen ion concentration of an acid with a **pH=4.2**?

Conclusion

Logarithmic functions are the inverses of exponential functions. A logarithm is defined as: $y = \log_a x$ if and only if $x = a^y$. The domain of a logarithmic function is limited to positive values of '**x**'. Logarithms are used in a variety of applications including compound interest, pH of chemicals, and magnitudes of earthquakes.