Notes on Matrices
4-1-2

Definition of a Matrix

Element

**A matrix can be named using its dimensions.

Dimension

Examples:

1. $A = \begin{bmatrix} 2 & -1 \\ 0 & 5 \\ -4 & 8 \end{bmatrix}$

2. $B = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$

3. $C = \begin{bmatrix} 0 & 5 & 3 & -1 \\ -2 & 0 & 9 & 6 \end{bmatrix}$

Row Matrix

Column Matrix

Square Matrix

Using matrices to solve problems:

Jim, Mario and Mike are married to Shana, Kelly and Lisa. Mario is Kelly's brother and lives in Florida with his wife. Mike is shorter than Lisa's husband. Mike works at a bank. Shana and her husband live in Kentucky. Kelly and her husband work in a candy store. Who is married to whom? Find out using a matrix!

Matrices

Equal Matrices

Solve for *x* and *y*.

1. $\begin{bmatrix} 2x \\ 2x+3y \end{bmatrix} = \begin{bmatrix} y \\ 12 \end{bmatrix}$

2. $\begin{bmatrix} 3x+y \\ x-2y \end{bmatrix} = \begin{bmatrix} x+3 \\ y-2 \end{bmatrix}$

3. $\begin{bmatrix} 2x & 3 & 3z \end{bmatrix} = \begin{bmatrix} 5 & 3y & 9 \end{bmatrix}$

Adding and Subtracting Matrices

Only matrices with ________________________________ can be added or subtracted.

The resulting matrix has ____________________ dimensions.

Examples

1. $\begin{bmatrix} -2 & 0 & 4 \\ 3 & -10 & 12 \\ 3 & -2 & -2 \end{bmatrix} + \begin{bmatrix} -4 & 6 & 0 \\ -15 & 2 & -4 \\ 6 & 7 & 1 \end{bmatrix}$

2. $\begin{bmatrix} 3 & -4 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 6 & 8 \\ 0 & 2 & 4 \end{bmatrix}$

3. $\begin{bmatrix} 2 & -18 \\ 20 & -5 \end{bmatrix} - \begin{bmatrix} -4 & 2 \\ -5 & 1 \end{bmatrix}$

4. $\begin{bmatrix} -3 & 10 & 2 \\ -10 & 8 & -6 \\ 0 & 1 & 0 \end{bmatrix} - \begin{bmatrix} 5 & 4 & 3 \\ 2 & 1 & 0 \\ -8 & -10 & -4 \end{bmatrix}$

Matrices

Scalar Multiplication

Examples:

1. $-2\begin{bmatrix} 7 & -1 & 0 \end{bmatrix}$

2. $4\begin{bmatrix} -2 & 0 \\ 4 & -5 \end{bmatrix}$

3. $-5\begin{bmatrix} 1 & 2 & -3 \\ 4 & -5 & 6 \\ -7 & 8 & -9 \end{bmatrix}$

Matrix Multiplication

Multiply rows times columns
**You can only multiply if the number of columns in the 1st matrix is equal to the number of rows in the 2nd matrix.

1. $\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 3 & -9 & 2 \\ 5 & 7 & -6 \end{bmatrix}$

2. $\begin{bmatrix} 3 & -9 & 2 \\ 5 & 7 & 6 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$

3. $\begin{bmatrix} 1 & 2 & -1 \\ 1 & 3 & 2 \\ 2 & 6 & 1 \end{bmatrix}\begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 8 \end{bmatrix}$

4. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 4 & -3 \\ -2 & 5 \end{bmatrix}$

Matrices

Determinants

Determinant of a 2x2 matrix:

Find the determinant of each:

1. $\begin{vmatrix} -5 & -7 \\ 11 & 8 \end{vmatrix}$

2. $\begin{vmatrix} 3 & 2 \\ -1 & 5 \end{vmatrix}$

3. $\begin{vmatrix} 10 & -2 \\ 0 & -3 \end{vmatrix}$

**To find a determinant you must have a _________ matrix!!

Determinant for a 3x3 matrix: Expansion by minors
*minor of an element is the determinant formed when the row and the column containing that element are deleted!

Examples:

1. $\begin{vmatrix} -2 & 3 & 8 \\ 6 & 7 & -1 \\ -4 & 5 & 9 \end{vmatrix}$

2. $\begin{vmatrix} 5 & -1 & 2 \\ 2 & -3 & 5 \\ 3 & 2 & -3 \end{vmatrix}$

3. $\begin{vmatrix} -1 & 0 & 4 \\ 2 & -2 & 2 \\ 3 & 0 & -1 \end{vmatrix}$

Matrices

Determinant for a 3x3 matrix: Diagonal Method
Examples:

1. $\begin{vmatrix} -2 & 3 & 8 \\ 6 & 7 & -1 \\ -4 & 5 & 9 \end{vmatrix}$

2. $\begin{vmatrix} 5 & -1 & 2 \\ 2 & -3 & 5 \\ 3 & 2 & -3 \end{vmatrix}$

3. $\begin{vmatrix} -1 & 0 & 4 \\ 2 & -2 & 2 \\ 3 & 0 & -1 \end{vmatrix}$

Solving Systems using Cramer's Rule

Cramer's Rule:

Identity and Inverse Matrices

Identity Matrix

Inverse of a 2x2 matrix

Examples:

1. $\begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

3. $\begin{bmatrix} 2 & -5 \\ 0 & 7 \end{bmatrix}$

???Is there ever a square matrix that does not have an inverse???

Matrices

Solving Systems of equations using matrices

- Coefficient Matrix
- Variable Matrix
- Constant Matrix

Example: $\begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix} \bullet \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \end{bmatrix}$

Steps to solving:

1.

2.

3.

Example: Solve using the inverse matrix method

$4x - 12y = 7$
$x + 6y = 9$