

Complex Numbers

Definition of Pure imaginary numbers

- $i = \sqrt{-1}$
- $i^2 = -1$

** i is not a variable – it is a symbol for a specific number **

Simplify each expression.

1. $\sqrt{-81}$

2. $\sqrt{-121x^5}$

3. $\sqrt{-200x}$

4. $8i - 3i$

5. $\sqrt{-5} \sqrt{-20}$

Cycle of i

Use properties of powers to simplify.

6. i^{12}

7. i^{17}

8. i^{26}

9. i^{11}

Definition of a complex number

Definition of equal complex numbers

Examples: Simplify

11. $(8 + 7i) + (-12 + 11i)$

12. $(9 - 6i) - (12 + 2i)$

13. $(8 + 5i)(2 - 3i)$

14. $(-6 + 2i)(5 - 3i)$

Real life application – Electricity!

The Habitat for humanity project utilizes volunteers to help build house for low – income families who might not be able to afford the purchase of a home. At a recent site, Habitat workers built a small storage shed attached to the house. The electrical blueprint for the shed called for two AC circuits connected in series with a total voltage of 220 volts. One of the circuits must have an impedance of $7 - 10j$ ohms, and the other needs to have an impedance of $9 + 5j$ ohms. According to the building codes, the impedance cannot exceed $20 - 5j$ ohms. Will the circuits, as designed, meet the code?

The total impedance is the sum of the individual impedance.

Complex conjugates

Examples: $1 + 8i$

$2 - i$

Simplify each:

1. $(6 + 3i)(6 - 3i)$

2. $(9 - 7i)(9 + 7i)$

3. $\frac{2 + i}{5i}$

4. $\frac{3 + 7i}{2i}$

5. $\frac{3i}{2 - 4i}$

6. $\frac{8i}{1 + 3i}$

Application – Fractal pictures created with the help of computers. Fractal objects are created using functions that are iterated, that is, repeated over and over. The function is evaluated for some initial value of x , and then that function is evaluated again using the result. Repeating this process and plotting the points produces interesting and beautiful pictures. Some of the most exciting fractals are created using complex numbers.

Suppose the function $f(x) = \frac{1}{1 - x^2}$ is to be iterated to produce a fractal. Find the first two points of the iteration if the initial value is $(i - 1)$.