You have probably done lots of graphing in Cartesian (x-y) coordinates, but relatively little in polar (r, \(\theta\)) coordinates. In this activity, you will explore various functions plotted in both coordinate systems. You will find connections between the two types of graphing and use your understanding of Cartesian graphing to better understand polar graphing.

### SKETCH AND INVESTIGATE

Start by investigating functions of the form \(f(t) = a\cos(bt)\). You can think of these as \(y = a\cos(bx)\) in the Cartesian plane and \(r = a\cos(b\theta)\) in the polar plane.

1. **Q1** Start with \(a = 5\) and \(b = 3\). Use what you know about graphing in the Cartesian plane to make an approximate sketch of \(y = 5\cos(3x)\) on the following grid.

   ![Sketch Grid]

   What do the parameters \(a\) and \(b\) control?

2. **Q2** In a moment, you’ll reveal the corresponding polar curve, \(r = 5\cos(3\theta)\). First, make a wild guess about what it will look like, and write down your guess.

3. **Q3** After you have made a guess, press the Show Polar button. Compare the result with your guess.
5. Drag the purple pointer back and forth slowly, this time looking for connections between the Cartesian and polar graphs. Note that the “bowtie” is always in the same relative position on the two output axes, but the red bar corresponding to $x$ in the Cartesian plane slides right and left, and the red bar corresponding to $\theta$ in the polar plane spins around.

Q3 Which points on the polar graph correspond to $x$-intercepts on the Cartesian graph?

Q4 Which points on the polar graph correspond to maximum points on the Cartesian graph? Which points on the polar graph correspond to minimum points on the Cartesian graph? Is there any connection between these points?

Q5 Dragging the purple pointer from $0^\circ$ to $360^\circ$, the polar graph repeats itself, with an interesting twist. What is different about the second copy?

Q6 Make a prediction as to how changing parameter $a$ will affect the polar graph. Now adjust parameter $a$ in the sketch, and see if you’re right. What does the $a$-value appear to control in the polar sketch?

Q7 Make a prediction as to how changing parameter $b$ will affect the polar graph. Adjust parameter $b$ in the sketch, and see if you’re right. What does the $b$-value appear to control in the polar sketch? Be sure to try both odd and even values.

EXPLORE MORE

Q8 Explain the pattern you discovered in Q7. (Hint: Your answer to Q5 is relevant here. Think about what has to be true in order for the polar graph to repeat itself at $180^\circ$ and for which values of $b$ this can happen.)

Q9 Go to page 2 of Cartesian Polar.gsp. There you’ll see $f(x) = \frac{2}{\cos(x)}$, which you could also write as $f(x) = 2\sec(x)$. Predict what the polar graph will look like. After you’ve recorded your prediction, press the Show Polar button to reveal the answer. Can you explain analytically why the graph looks this way?

Q10 Go to page 3 of Cartesian Polar.gsp. On this page you can enter your own functions to try them out. You can use the values of parameters $a$ and $b$ in these functions if you like. Try several different kinds of functions, and record your results.
**CARTESIAN GRAPHS AND POLAR GRAPHS**  
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**Objective:** Students compare rectangular graphs and polar graphs for functions in the form \( y = a \sin(bx) \) and \( r = a \sin(b \theta) \). Students analyze how the period and amplitude of a Cartesian graph correlate with features of the corresponding polar graph.

**Prerequisites:** Students should be familiar with the graphs of sinusoids, and specifically with the concepts of period and amplitude. This activity is best utilized shortly after students have been introduced to graphing in polar coordinates.

**Sketchpad Proficiency:** Beginner. Students manipulate a pre-made sketch.

**Class Time:** 20–30 minutes

**Required Sketch:** Cartesian Polar.gsp

**SKETCH AND INVESTIGATE**

Q1 The parameter \( a \) controls the amplitude of the sinusoid. The parameter \( b \) controls the period.

Q2 Answers will vary. The point of this question is to get students thinking about the process before they reveal the answer. Students have no basis yet for guessing correctly, so emphasize that the important thing is guessing, not getting the answer right.

Q3 The \( x \)-intercepts on the Cartesian graph correspond to \( r = 0 \), where the curve crosses the pole (origin) of the polar graph.

Q4 The maximum and minimum points on the Cartesian graph correspond to the outermost point of each leaf of the polar graph. The only difference between the two types of points on the polar graph is that the maximum points are created when the output value \( (r) \) is positive (the “bowtie” is on the positive side of the output bar), and the minimum points are created when the output value is negative (the “bowtie” is on the negative side of the output bar).

Q5 The polar graph starts on the positive side of the output axis for the first repetition (starting at \( 0^\circ \)) and starts on the negative side of the output axis for the second repetition (starting at \( 180^\circ \)). At all corresponding points (points whose \( \theta \)-values are separated by \( 180^\circ \)), \( r \)-values are opposite.

Q6 The parameter \( a \) controls the distance from the pole to the tip of each leaf.

Q7 When the parameter \( b \) is odd, the number of leaves is equal to \( b \). When \( b \) is even, the number of leaves is equal to \( 2b \).

**EXPLORE MORE**

Q8 Between \( 0^\circ \) and \( 360^\circ \) (not including \( 360^\circ \)), the graph of \( y = a \cos(bx) \) has \( b \) maximums and \( b \) minimums for a total of \( 2b \) extreme points—the points that become the outer points on the leaves. For even integer values of the parameter \( b \), the Cartesian graph yields a maximum \( y \)-value at \( \theta = 0^\circ \) and \( \theta = 180^\circ \), meaning that the graph doesn’t double back on itself and there will be the maximum \( (2b) \) number of leaves. For odd integer values of the parameter \( b \), the Cartesian graph yields a maximum \( y \)-value at \( \theta = 0^\circ \), but it yields a minimum at \( \theta = 180^\circ \), causing the graph to double over itself at \( 180^\circ \) and resulting in only \( b \) leaves.

Q9 This equation produces a vertical line 2 units to the right of the pole. Students can use trigonometry to explain why this occurs. Consider the right triangle in the following diagram, using an arbitrary point \( B \) on the vertical line through point \( A \) at \( (2, 0^\circ) \).
To find the length of the hypotenuse OB, use the cosine function:

\[ \cos(\theta) = \frac{OA}{OB} = \frac{2}{OB} \]

\[ OB = \frac{2}{\cos(\theta)} = 2 \sec(\theta) \]

Thus the equation of any point on the vertical line must be \( r = 2 \sec(\theta) \).

Q10 Three interesting functions to try are

\[ f(\theta) = a \tan(b \theta), \]
\[ f(\theta) = a \left( \frac{\theta}{90^\circ} \right), \]
\[ \text{and } f(\theta) = a \sqrt{\frac{\theta}{90^\circ}} \]