Direct and Indirect Variation

Variation, in general, will concern two variables; say height and weight of a person, and how, when one of these changes, the other might be expected to change.

- We have **direct variation** if the two variables change in the same sense; i.e. if one increases, so does the other.
- We have **indirect variation** if one going up causes the other to go down. An example of this might be speed and time to do a particular journey; so the higher the speed, the shorter the time.

Normally we let x be the independent variable, and y the dependent variable, so that a change in x produces a change in y. For example, if x is number of motor cars on the road, and y the number of accidents; then we expect an increase in x to cause an increase in y. (This obviously ceases to apply if number of cars is so large that they are all stationary in a traffic jam.)

Direct Variation

When x and y are directly proportional, then doubling x will double the value of y; and if we divide these variables we get a constant result. Since if $\frac{y}{x} = k$ then $\frac{2y}{2x} = k$ where k is called the **constant of proportionality.**

We could also write this $y = kx$. Thus if I am given the value of x, I multiply this number by k to find the value of y.

Example: Given that y and x are directly proportional, and $y = 2$ when $x = 5$, find the value of when $x = 15$.

We first find value of k, using $\frac{y}{x} = k$. $\rightarrow \frac{2}{5} = k$

Now use this constant value in the equation $y=kx$ *for situation when* $x = 15$.

$$y = \frac{2}{5} \bullet 15 \rightarrow = \frac{30}{5} = 6$$

If you want to do this quickly in your head, you could say x has been multiplied by a factor 3 (going from 5 to 15), so y must also go up by a factor of 3. That means y goes from 2 to 6.

Direct and Indirect Variation

Indirect Variation.

We gave an example of inverse proportion above, namely speed and time for a particular journey.

In this case, if you double the speed, you halve the time. So the product, speed *x* time = constant.

In general, if x and y are inversely proportional, then the product xy will be constant.

$$xy = k \text{ or } y = \frac{k}{x}$$

Example: If it takes 4 hours at an average speed of $90\frac{km}{hr}$ to do a certain journey, how long would it take at $120\frac{km}{hr}$?

k = speed • time = 90 • 4 = 360 (k in this case is the distance.)

Then time $= \frac{k}{speed} = \frac{360}{120} =$ 3 hours.

To do this in your head, you could say that speed has changed by a factor $\frac{3}{4}$, so time must change by a factor, $\frac{3}{4}$. However, for the usual type of problem, go through the steps I outlined above.

I hope these examples have made the idea of variation (both direct and inverse) reasonably clear.

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