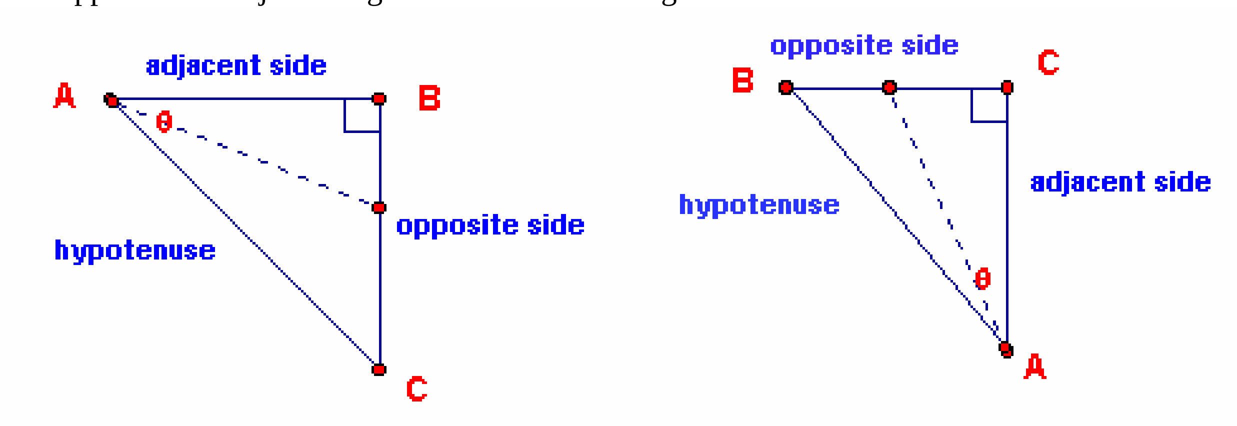
Trigonometry

Angles & Circular Functions: Solving Right Triangles

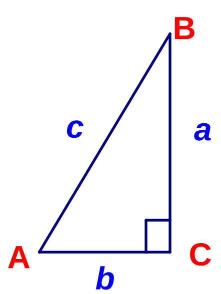
Trigonometric Functions in a Right Triangle

We have already looked at the trigonometric functions from the perspective of the ***unit circle***. Now we will explore the classical model of trigonometry, which is based on the relationships between angles and sides in a ***right triangle***.

In this section we will use terms like ***opposite leg, adjacent leg*** and ***hypotenuse.*** The hypotenuse of a right triangle is always the longest side, directly across from the right angle. The opposite and adjacent legs are shown in the diagram below.



For angle A in right triangle ABC below, the six trigonometric functions are defined as:



$$\sin A = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{a}{c} \qquad \csc A = \frac{\text{hypotenuse}}{\text{opposite side}} = \frac{c}{a}$$

$$\cos A = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{b}{c} \qquad \sec A = \frac{\text{hypotenuse}}{\text{adjacent side}} = \frac{c}{b}$$

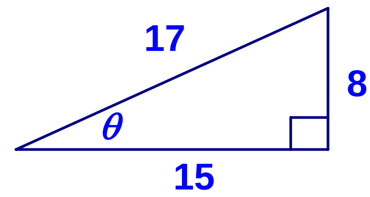
$$\tan A = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{a}{b} \qquad \cot A = \frac{\text{adjacent side}}{\text{opposite side}} = \frac{b}{a}$$

A useful way of remembering this is the mnemonic device: **SOH - CAH - TOA**
which stands for:

SOH $\qquad$ **CAH** $\qquad$ **TOA**

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

Example: A right triangle has sides whose lengths are 8 in., 15 in. and 17 in. Find the values of the six trig functions of the acute angle, θ, shown in the diagram.



$$\sin\theta = \frac{opposite}{hypotenuse} = \frac{8}{17} \qquad \csc\theta = \frac{hypotenuse}{opposite} = \frac{17}{8}$$

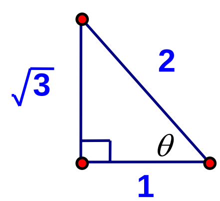
$$\cos\theta = \frac{adjacent}{hypotenuse} = \frac{15}{17} \qquad \sec\theta = \frac{hypotenuse}{adjacent} = \frac{17}{15}$$

$$\tan\theta = \frac{opposite}{adjacent} = \frac{8}{15} \qquad \cot\theta = \frac{adjacent}{opposite} = \frac{15}{8}$$

Try These:

1. A right triangle has sides whose lengths are 3 ft, 4 ft and 5 ft. Find the values of the six trigonometric functions of the smallest angle.

2. Evaluate the six trigonometric functions for θ in the right triangle shown below.

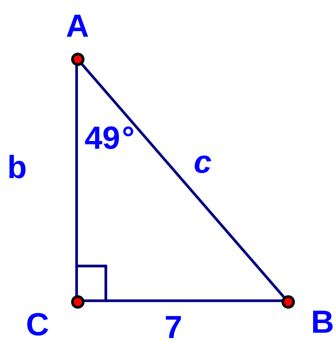


Solving Right Triangles

Now that we know how to *write* the six trigonometric functions of a right triangle we can use these ratios to *solve* a right triangle. Solving a right triangle means that we use given side and angle measures to calculate missing side and angle measures.

Make sure your calculator is set in degree mode or the angle will be given in radians!

Example 1: Solve right triangle ABC shown below. Round angle measures to the nearest degree and side measures to the nearest tenth.



Solution:
We know that the three angles of this triangle add up to 180°, so:

$$\angle B = 180^\circ - 49^\circ - 90^\circ = 41^\circ$$

Next we set up some trigonometric functions. Since the given side, BC, is *opposite* the given angle, 49°, we need trig functions that use the *opposite* side:

$\sin 49^\circ = \frac{7}{c}$ and $\tan 49^\circ = \frac{7}{b}$

$c \sin 49^\circ = \frac{7}{c} \bullet c$ $\quad$ $b \tan 49^\circ = \frac{7}{b} \bullet b$

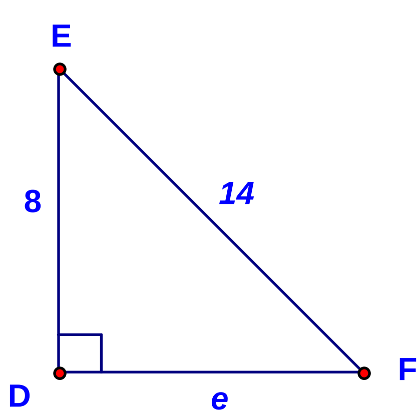
$c \sin 49^\circ = 7$ $\quad$ $b \tan 49^\circ = 7$

$c = \frac{7}{\sin 49^\circ}$ $\quad$ $b = \frac{7}{\tan 49^\circ}$

$c \approx 9.3$ $\quad$ $b \approx 6.1$

In the next example, we are only given the side measures, and we must find the acute angles. To do this we use the $\sin^{-1}$ *which can be found on your calculator using the 2nd or Shift button.*

Example 2: Solve right triangle ΔDEF. Round angle measures to the nearest degree and side measures to the nearest tenth.



Solution:
First we will find $\angle$ F using its opposite side (8) and the hypotenuse (14):

$$\sin F = \frac{8}{14}$$

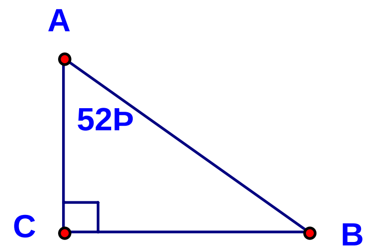
$$\angle F = \sin^{-1}\frac{8}{14} = 34.85^\circ \approx 35^\circ$$

$$\angle E = 180^\circ - 90^\circ - 35^\circ = 55^\circ$$

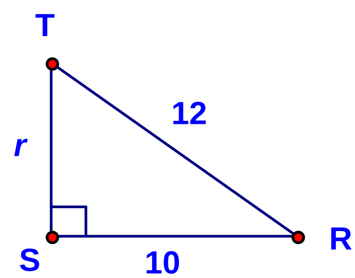
e: $\cos 35^\circ = \frac{e}{14}$, $e = 14 \cos 35^\circ \approx 11.5$

Try These:

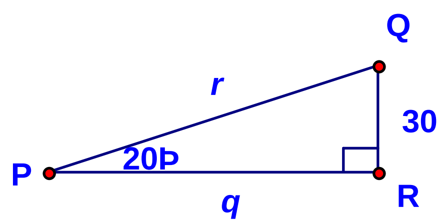
1. Solve right triangle ABC. Round angle measures to the nearest degree and side measures to nearest tenth.



2. Solve right triangle RST. Round angle measures to the nearest degree and side measures to nearest tenth.



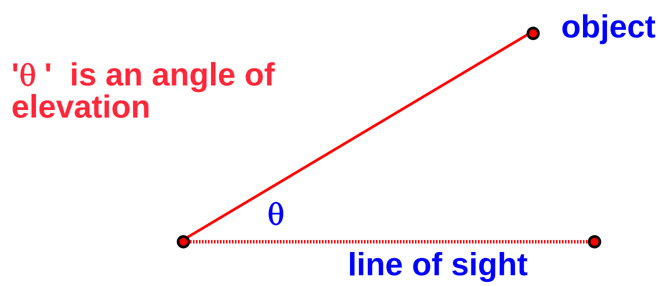
3. Solve right triangle PQR. Round angle measures to the nearest degree and side measures to nearest tenth.



Angle of Elevation, Angle of Depression

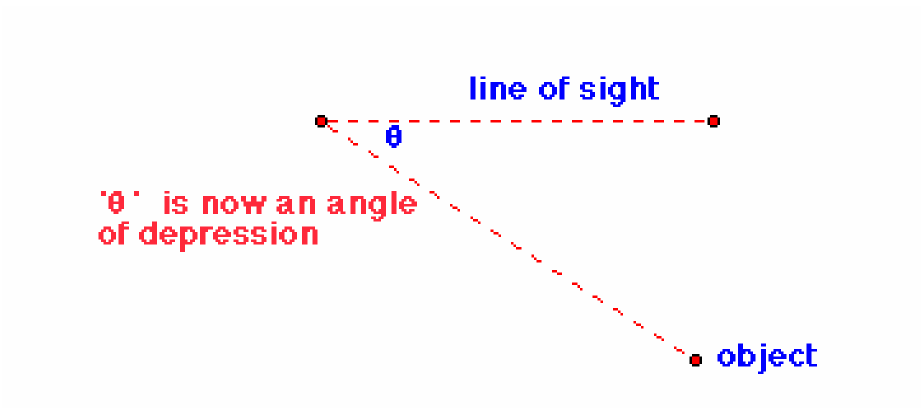
A. *Angle of Elevation*

This is the angle between a horizontal line of sight and the line of site to an object at a higher elevation.

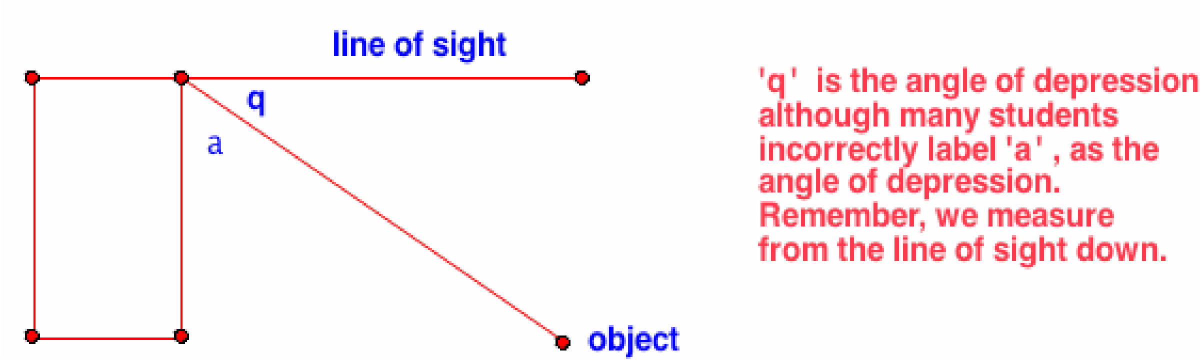


B. *Angle of Depression*

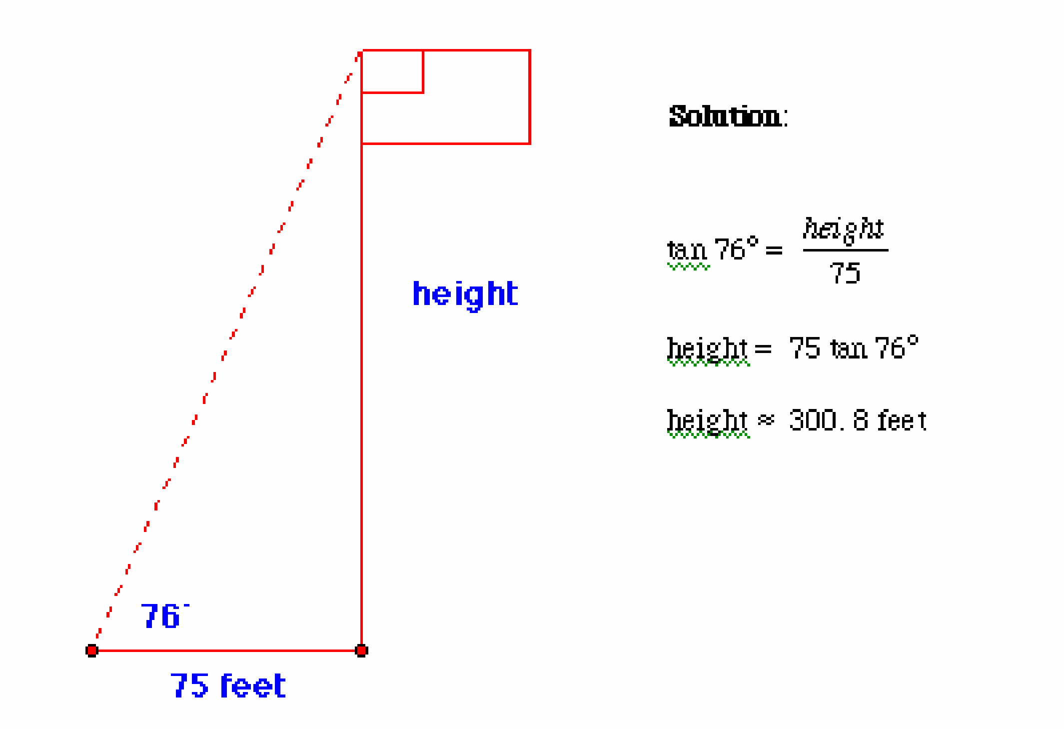
This is the angle between a horizontal line of sight and the line of site to an object at a lower elevation.



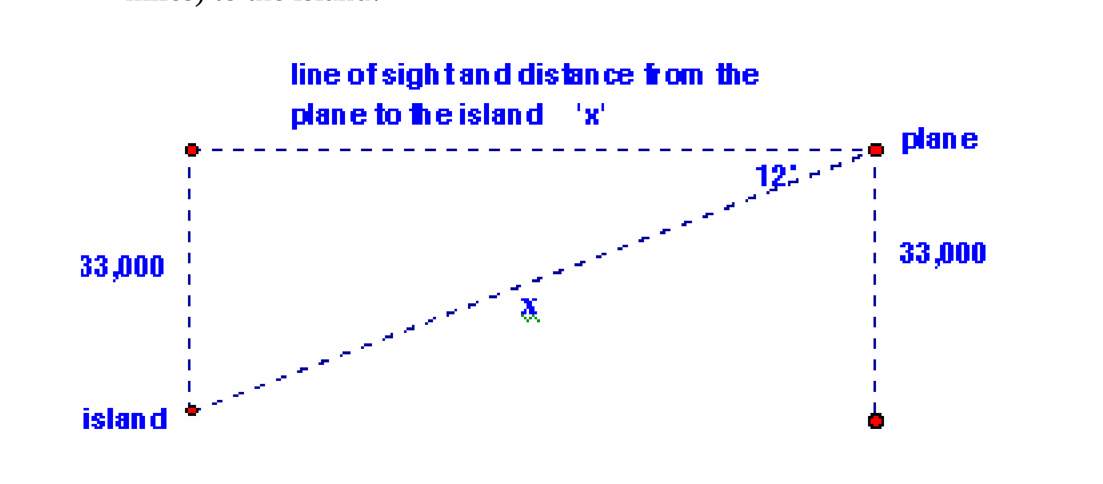
*** We must be careful when forming the angle of depression.** See the diagram below:



Example 1: A surveyor is standing 75 feet from the base of a large flagpole. The surveyor measures the angle of elevation to the top of the pole as 76˚. How tall is the flagpole?



Example 2: A navigator aboard a passenger jet measures the angle of depression to a tiny island to be 12˚. The line of sight from the plane to the island is x miles. If the jet is flying at an altitude of 33,000 feet, then what is the horizontal distance (in miles) to the island?



Solution: $\sin 12^\circ = \dfrac{33{,}000}{x}$

$x \sin 12^\circ = 33{,}000$

$x = \dfrac{33{,}000}{\sin 12^\circ} \approx 158721.2\ \textit{feet}$ or $\dfrac{158721.2}{5{,}280} \approx 30.1\ \textit{miles}$

Try These:

1. You are standing at the base of a tall television tower. You pace off 25 feet from the base of the tower. Looking up, you approximate the angle of elevation to the top of the tower to be 80˚. What is the approximate height of the tower?

2 You are building some wooden steps to reach your second story deck. If the height of the deck is to be 20 feet and the angle of elevation to the top is 36˚, then what will the length of the steps be?