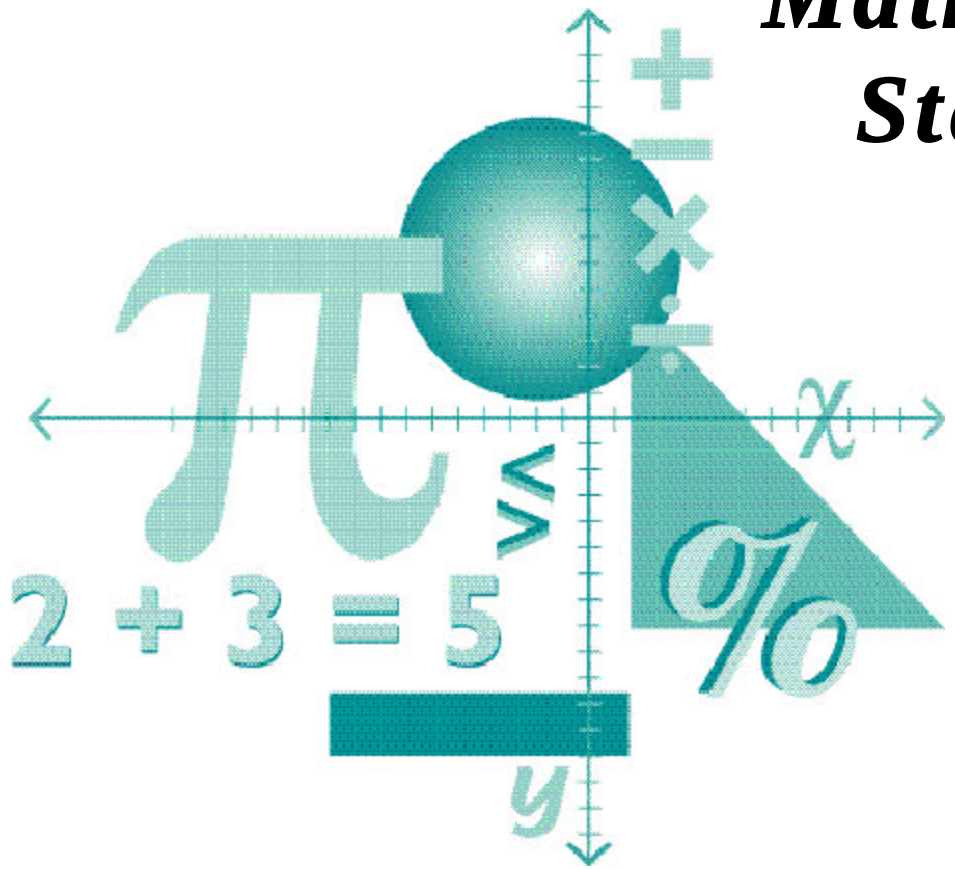

Mathematics Standards of Learning Curriculum Framework

Algebra I

Commonwealth of Virginia
Board of Education
Richmond, Virginia
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Introduction

Mathematics content develops sequentially in concert with a set of processes that are common to different bodies of mathematics knowledge. The content of the Mathematics Standards of Learning supports five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. These goals provide a context within which to develop the knowledge and skills identified in the standards.

Algebra provides a systematic way to represent mathematical relationships and analyze change. Students need to understand the concepts and symbols of algebra, the structures that govern the manipulation of the symbols, and how the symbols can be used to record ideas and events. Students should explore patterns that are linear and quadratic in the first year of algebra and should develop the notion of families of functions. A solid conceptual foundation in algebra should be developed before students engage extensively in symbolic manipulation.

Each topic in the Algebra I Curriculum Framework is developed around the Standards of Learning. Each Standard of Learning is expanded in the Essential Knowledge and Skills column. The Essential Understandings column includes concepts, mathematical relationships, and ideas that are important to understanding and teaching the Standard of Learning effectively.

Teachers should help students make connections and build relationships among algebra, arithmetic, geometry, discrete mathematics, and probability and statistics. Connections should be made to other subject areas and fields of endeavor through applications. Using manipulatives, graphing calculators, and computer applications to develop concepts should help students develop and attach meaning to abstract ideas. Throughout the study of mathematics, students should be encouraged to talk about mathematics, use the language and symbols of mathematics, communicate, discuss problems and problem solving, and develop their competence and their confidence in themselves as mathematics students.

TOPIC: EXPRESSIONS AND OPERATIONS

ALGEBRA I
STANDARD A.2

The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables. Students will choose an appropriate computational technique, such as mental mathematics, calculator, or paper and pencil.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Algebra is a tool for reasoning about quantitative situations so that relationships become apparent. • Algebra is a tool for describing and representing patterns and relationships. • The numerical values of an expression are dependent upon the values of the replacement set for the variables. • There are a variety of ways to compute the value of a numerical expression and evaluate an algebraic expression. • The operations and the magnitude of the numbers in an expression impact the choice of an appropriate method of computation.	• Translate verbal expressions into algebraic expressions with three or fewer terms. • Relate a polynomial expression with three or fewer terms to a verbal expression. • Evaluate algebraic expressions for a given replacement set to include integers and rational numbers. • Apply appropriate computational techniques to evaluate an algebraic expression.

TOPIC: EXPRESSIONS AND OPERATIONS

ALGEBRA I
STANDARD A.10

The student will apply the laws of exponents to perform operations on expressions with integral exponents, using scientific notation when appropriate.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Repeated multiplication can be represented with exponents. • The laws of exponents can be investigated using patterns. • The base and the exponent impact the magnitude of the expression. • A relationship exists between the laws of exponents and scientific notation.	• Identify the base, exponent, and coefficient in a monomial expression. • Simplify monomial expressions and ratios of monomial expressions in which the exponents are integers, using the laws of exponents. • Express numbers, using scientific notation, and perform operations, using the laws of exponents.

TOPIC: EXPRESSIONS AND OPERATIONS

ALGEBRA I
STANDARD A.11

The student will add, subtract, and multiply polynomials and divide polynomials with monomial divisors, using concrete objects, pictorial and area representations, and algebraic manipulations.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A relationship exists between arithmetic operations and operations with polynomials. • Polynomials can be represented in a variety of forms. • Operations with polynomials can be represented concretely, pictorially, and algebraically. • Polynomial expressions can be used to model real-life situations. • The distributive property is the unifying concept for polynomial operations.	• Model sums, differences, products, and quotients of polynomials with concrete objects and their related pictorial representations. • Relate concrete and pictorial representations for polynomial operations to their corresponding algebraic manipulations. • Find sums and differences of polynomials. • Multiply polynomials by monomials and binomials by binomials symbolically. • Find the quotient of polynomials, using a monomial divisor.

TOPIC: EXPRESSIONS AND OPERATIONS

ALGEBRA I
STANDARD A.12

The student will factor completely first- and second-degree binomials and trinomials in one or two variables. The graphing calculator will be used as a tool for factoring and for confirming algebraic factorizations.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Factoring reverses polynomial multiplication. • There is a relationship between the factors of a polynomial and the x-intercepts of its related graph. • Some polynomials cannot be factored over the set of real numbers. • Polynomial expressions in a variable x and their factors can be used to define functions by setting y equal to the polynomial expression or y equal to a factor, and these functions can be represented graphically.	• Use the distributive property to "factor out" all common monomial factors. • Factor second-degree polynomials and binomials with integral coefficients and a positive leading coefficient less than four. • Identify polynomials that cannot be factored over the set of real numbers. • Use the x-intercepts from the graphical representation of the polynomial to determine and confirm its factors.

TOPIC: EXPRESSIONS AND OPERATIONS

ALGEBRA I
STANDARD A.13

The student will express the square root of a whole number in simplest radical form and approximate square roots to the nearest tenth.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The square root of a perfect square is an integer. • The square root of a non-perfect square lies between two consecutive integers. • The inverse of squaring a number is determining the square root. • A radical in simplest form is one in which the radicand has no perfect square factors other than one. • The square root of a product is the product of the square roots.	• Estimate the square root of a non-perfect square to the nearest tenth by – identifying the two perfect squares it lies between; – finding the square root of those two perfect squares; and – using those values to estimate the square root of the non-perfect square. • Find the square root of a number, and make a reasonable interpretation of the displayed value for a given situation, using a calculator. • Express the square root of a whole number less than 1,000 in simplest radical form.

TOPIC: RELATIONS AND FUNCTIONS

ALGEBRA I
STANDARD A.5

The student will create and use tabular, symbolic, graphical, verbal, and physical representations to analyze a given set of data for the existence of a pattern, determine the domain and range of relations, and identify the relations that are functions.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A set of data may be characterized by patterns, and those patterns can be represented in multiple ways. • Graphs can be used as visual representations to investigate relationships between quantitative data. • Algebra is a tool for describing patterns, making generalizations, and representing a relationship in which output is related to input. • A function is a relation for which there is a unique output for each input. • A relation can be represented by a set of ordered pairs. • The domain consists of the first coordinates of the ordered pairs. • The range consists of the second coordinates of the ordered pairs. • A relation is a function if each element in the domain is paired with a unique element of the range.	• Analyze a table of ordered pairs for the existence of a pattern that defines the change relating input and output values. • Write a linear equation to represent a pattern in which there is a constant rate of change between variables. • Determine from a set of ordered pairs, a table, or a graph whether a relation is a function. • Identify the domain and range for a relation, given a set of ordered pairs, a table, or a graph. • Use physical representations, such as algebra manipulatives, to represent quantitative data.

TOPIC: RELATIONS AND FUNCTIONS

ALGEBRA I
STANDARD A.15

The student will, given a rule, find the values of a function for elements in its domain and locate the zeros of the function both algebraically and with a graphing calculator. The value of $f(x)$ will be related to the ordinate on the graph.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• An equation represents the relationship between the independent and dependent variables. • The object $f(x)$ is the unique object in the range of the function f that is associated with the object x in the domain of f. • For each x in the domain of f, x is a member of the input of the function f, $f(x)$ is a member of the output of f, and the ordered pair $[x, f(x)]$ is a member of f. • An object x in the domain of f is an x-intercept or a zero of a function f if and only if $f(x) = 0$.	• For each x in the domain of f, find $f(x)$. • Identify the zeros of the function algebraically and confirm them, using the graphing calculator.

TOPIC: RELATIONS AND FUNCTIONS

ALGEBRA I
STANDARD A.18

The student will analyze a relation to determine whether a direct variation exists and represent it algebraically and graphically, if possible.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The constant k in a direct variation is represented by the ratio of y to x ($k = \frac{y}{x}$), where y is the dependent variable. • Direct variation is used to represent a constant rate of change in practical applications. • In direct variation, equal changes in x result in proportional changes in y.	• Given a table of values, determine whether a direct variation exists. • Write an equation for a direct variation, given a set of data. • Graph a direct variation from a table of values or a practical situation.

TOPIC: EQUATIONS AND INEQUALITIES

ALGEBRA I
STANDARD A.1

The student will solve multistep linear equations and inequalities in one variable, solve literal equations (formulas) for a given variable, and apply these skills to solve practical problems. Graphing calculators will be used to confirm algebraic solutions.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A solution to an equation or inequality is the value or set of values that can be substituted to make the equation or inequality true. • Equations and inequalities can be solved in a variety of ways. • The solution of an equation in one variable can be found by graphing each side of the equation separately and finding the x-coordinate of the point of intersection. • Practical problems can be interpreted, represented, and solved using linear equations and inequalities in one variable.	• Translate verbal sentences to algebraic equations and inequalities in one variable. • Solve multistep linear equations and noncompound inequalities in one variable with the variable in both sides of the equation or inequality. • Solve multistep linear equations and inequalities in one variable with grouping symbols in one or both sides of the equation or inequality. • Solve multistep equations and inequalities in one variable with rational coefficients and constants. • Solve a literal equation (formula) for a specified variable. • Apply skills for solving linear equations to practical situations. • Confirm algebraic solutions to linear equations and inequalities, using a graphing calculator.

TOPIC: EQUATIONS AND INEQUALITIES

ALGEBRA I
STANDARD A.3

The student will justify steps used in simplifying expressions and solving equations and inequalities. Justifications will include the use of concrete objects; pictorial representations; and the properties of real numbers, equality, and inequality.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The representation and manipulation of expressions, equations, and inequalities can be modeled in a variety of ways, using concrete, pictorial, and symbolic representations. • Properties of real numbers and properties of equations and inequalities can be used to solve equations and inequalities and simplify expressions.	• Simplify expressions and solve equations and inequalities, using the commutative, associative, and distributive properties. • Simplify expressions and solve equations and inequalities, using the order of operations. • Solve equations, using the addition, multiplication, closure, identity, and inverse properties. • Solve equations, using the reflexive, symmetric, transitive, and substitution properties of equality. • Create and interpret pictorial representations for simplifying expressions and solving equations and inequalities.

TOPIC: EQUATIONS AND INEQUALITIES

ALGEBRA I
STANDARD A.6

The student will select, justify, and apply an appropriate technique to graph linear functions and linear inequalities in two variables. Techniques will include slope-intercept, *x*- and *y*-intercepts, graphing by transformation, and the use of the graphing calculator.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Linear functions and inequalities can be written in a variety of forms. • Linear functions and inequalities can be graphed, using a variety of techniques. • An appropriate technique for graphing linear functions and inequalities can be determined by the given information and/or the tools available. • Justification of an appropriate technique for graphing linear equations and inequalities is dependent upon the application of slope, *x*- and *y*-intercepts, and graphing by transformations. • Linear equations and inequalities arise from a variety of practical situations.	• Graph linear equations and inequalities in two variables that arise from a variety of practical situations. • Use the line $y = x$ as a reference, and apply transformations defined by changes in the slope or *y*-intercept. • Express linear functions or inequalities in slope-intercept form, and use the graphing calculator to display the relationship. • Explain why a given technique is appropriate for graphing a linear function.

TOPIC: EQUATIONS AND INEQUALITIES

ALGEBRA I
STANDARD A.7

The student will determine the slope of a line when given an equation of the line, the graph of the line, or two points on the line. Slope will be described as rate of change and will be positive, negative, zero, or undefined. The graphing calculator will be used to investigate the effect of changes in the slope on the graph of the line.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The slope of a linear function represents a constant rate of change in the dependent variable when the independent variable changes by a fixed amount. • The slope of a line determines its relative steepness. • The slope of a line can be determined in a variety of ways. • Changes in slope affect the graph of a line.	• Recognize that m represents the slope in the equation of the form $y = mx + b$. • Find the slope of the line, given the equation of a linear function. • Calculate the slope of a line, given the coordinates of two points on the line. • Find the slope of a line, given the graph of a line. • Recognize and describe a line with a slope that is positive, negative, zero, or undefined. • Describe slope as a constant rate of change between two variables. • Compare the slopes of graphs of linear functions, using the graphing calculator.

TOPIC: EQUATIONS AND INEQUALITIES

ALGEBRA I
STANDARD A.8

The student will write an equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The equation of a line defines the relationship between two variables. • The graph of a line represents the set of points that satisfies the equation of a line. • A line can be represented by its graph or by an equation. • The equation of a line can be determined by two points on the line or by the slope and a point on the line.	• Recognize that equations of the form $y = mx + b$ and $Ax + By = C$ are equations of lines. • Write an equation of a line when given the graph of a line. • Write an equation of a line when given two points on the line whose coordinates are integers. • Write an equation of a line when given the slope and a point on the line whose coordinates are integers. • Write an equation of a vertical line as $x = c$. • Write an equation of a horizontal line as $y = c$.

TOPIC: EQUATIONS AND INEQUALITIES

ALGEBRA I
STANDARD A.9

The student will solve systems of two linear equations in two variables both algebraically and graphically and apply these techniques to solve practical problems. Graphing calculators will be used both as a primary tool for solution and to confirm an algebraic solution.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A system of linear equations with exactly one solution is characterized by the graphs of two lines whose intersection is a single point, and the coordinates of this point satisfy both equations. • A point shared by two intersecting graphs and the ordered pair that satisfies the equations characterize a system of equations with only one solution. • A system of two linear equations with no solution is characterized by the graphs of two lines that do not intersect but are parallel. • A system of two linear equations having infinite solutions is characterized by two graphs that coincide (the graphs will appear to be the graph of one line), and all the coordinates on this one line satisfy both equations. • Systems of two linear equations can be used to represent two conditions that must be satisfied simultaneously.	• Given a system of two linear equations in two variables that has a unique solution, solve the system by substitution or elimination to find the ordered pair which satisfies both equations. • Given a system of two linear equations in two variables that has a unique solution, solve the system graphically to find the point of intersection. • Determine whether a system of two linear equations has one solution, no solution, or infinite solutions. • Write a system of two linear equations that describes a practical situation. • Interpret and determine the reasonableness of the algebraic or graphical solution of a system of two linear equations that describes a practical situation.

TOPIC: EQUATIONS AND INEQUALITIES

ALGEBRA I
STANDARD A.14

The student will solve quadratic equations in one variable both algebraically and graphically. Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The zeros or the *x*-intercepts of the quadratic function are the real root(s) or solution(s) of the quadratic equation that is formed by setting the given quadratic expression equal to zero. • Quadratic equations can be solved in a variety of ways. • A quadratic equation can have two solutions, one solution, or no solution. • A solution to a quadratic equation is the value or set of values that can be substituted to make the equation true.	• Solve quadratic equations algebraically or by using the graphing calculator. When solutions are represented in radical form, the decimal approximation will also be given. • Verify algebraic solutions, using the graphing calculator. • Identify the *x*-intercepts of the quadratic function as the solution(s) to the quadratic equation that is formed by setting the given quadratic expression equal to zero.

TOPIC: STATISTICS

ALGEBRA I
STANDARD A.4

The student will use matrices to organize and manipulate data, including matrix addition, subtraction, and scalar multiplication. Data will arise from business, industrial, and consumer situations.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Matrices are a tool for organizing and displaying data. • A relationship exists between arithmetic operations and operations with matrices. • Matrices can be used to solve practical problems. • Only matrices of the same dimensions can be added or subtracted.	• Represent data from practical problems in matrix form. • Calculate the sum or difference of two given matrices that are no larger than 4 x 4. • Calculate the product of a scalar and a matrix that is no larger than 4 x 4. • Solve practical problems involving matrix addition, subtraction, and scalar multiplication, using matrices that are no larger than 4 x 4. • Read and interpret the data in a matrix representing the solution to a practical problem.

TOPIC: STATISTICS

ALGEBRA I
STANDARD A.16

The student will, given a set of data points, write an equation for a line of best fit and use the equation to make predictions.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The graphing calculator can be used to determine the equation of a line of best fit for a set of data. • The line of best fit for the relationship among a set of data points can be used to make predictions where appropriate.	• Write an equation for the line of a best fit, given a set of six to ten data points in a table, on a graph, or from a practical situation. • Make predictions about unknown outcomes, using the equation of a line of best fit.

TOPIC: STATISTICS

ALGEBRA I
STANDARD A.17

The student will compare and contrast multiple one-variable data sets, using statistical techniques that include measures of central tendency, range, and box-and-whisker graphs.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Measures of central tendency can be used to characterize a set of data and to make predictions. • Statistical techniques can be used to organize, display, and compare sets of data. • Box-and-whisker plots can be used to analyze data.	• Calculate the measures of central tendency and range of a set of data with no more than 20 data points. • Compare measures of central tendency using numerical data from a table with no more than 20 data points. • Compare and contrast two sets of data, each set having no more than 20 data points, using measures of central tendency and the range. • Compare and analyze two sets of data, each set having no more than 20 data points, using box-and-whisker plots.