Advanced Algebra
Solving Polynomial Inequalities

<u>Interval Notation</u>
Solutions to inequalities can include infinite values. To be consistent, we will often write these solutions as intervals. Look at these sets of real numbers:

- All numbers between 5 and 6 … $5 < x < 6$ … $(5, 6)$
- All numbers between 5 and 6 including 5 and 6 … $5 \le x \le 6$ … $[5, 6]$
- All numbers between 5 and 6 including 6 … $5 < x \le 6$ … $(5, 6]$
- All numbers between 5 and 6 including 5 … $5 \le x < 6$ … $[5, 6)$
- All numbers larger than 3 (but less than infinity) … $x > 3$ … $(3, \infty)$
- All numbers greater than or equal to 3 (but less than infinity) … $x \ge 3$ … $[3, \infty)$
- All numbers less than 3 (but greater than negative infinity) … $x < 3$ … $(\infty, 3)$
- All numbers less than or equal to 3 (but greater than negative infinity) … $x \le 3$ … $(\infty, 3]$

To solve a polynomial inequality, you first need to find the real roots, or solutions, to the polynomial. You can factor or use the rational root theorem to find these.

Ex: Solve $x^3 + x^2 - 9x - 9 \le 0$

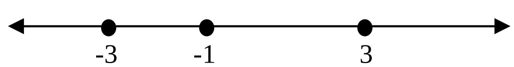
$$x^2(x+1) - 9(x+1) \le 0$$

$$(x^2 - 9)(x+1) \le 0$$

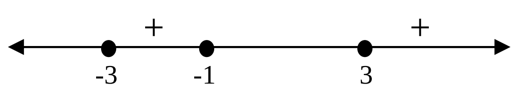
$$(x+3)(x-3)(x+1) \le 0$$

The real roots are x = -3, 3, -1

These values will be the endpoints of the solution interval(s). Plot them on a number line (also known as a sign chart). Since the inequality is $\le$, the endpoints will be included in the solution, so we plot them on the sign chart as solid dots. If the inequality did not include the endpoints, we would use open circles on the sign chart.



With three roots, our sign chart is divided into four intervals. Choose values for x in each interval and substitute into the polynomial. Mark the sign of the result on the sign chart.



Substitute x = -4 into $(x+3)(x-3)(x+1)$, get (-1)(-7)(-3) which is negative.
Substitute x = -2 into $(x+3)(x-3)(x+1)$, get (1)(-5)(-1) which is positive.
Substitute x = 0 into $(x+3)(x-3)(x+1)$, get (3)(-3)(1) which is negative.
Substitute x = 4 into $(x+3)(x-3)(x+1)$, get (7)(1)(5) which is positive.

Since the inequality is ≤ 0, the solution set contains the intervals that yield negative numbers, the solution is $(-\infty, -3] \cup [-1, 3]$.