Advanced Algebra
Polynomial Functions: Approximating Real Zeros of Polynomials

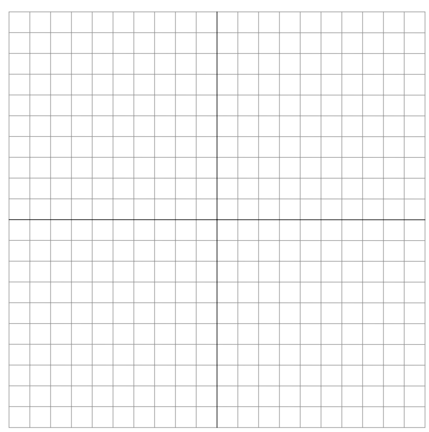
Polynomial functions can have *Real* or *Imaginary* roots. Real roots may be *Rational* or *Irrational*. The Rational Root Theorem and synthetic division can help us locate *Rational Real Roots*. Finding imaginary and irrational roots is more difficult. If the function is degree two or less (or can be factored such that each factor is degree two or less) we can use the Quadratic Formula to solve for any roots. If a function has a higher degree we can sometimes use the Rational Root Theorem and division to find the roots of the polynomial. Sometimes, there are no rational roots and none of these techniques work. In these cases we can use *The Location Principle* to *approximate* the value of *real* roots of the polynomial. [This technique does not help us find imaginary roots!]

Take the polynomial function: $f(x) = x^3 - 2x^2 + x - 3$

How many total roots does this function have? ___*3*__

Evaluate the function at each of the following *x*-values, fill in the table and plot the ordered pairs on the grid (don't connect the points yet).

x	**f(*x*)**
-3	
-2	
-1	
0	
1	
2	
3	



We know that this is a polynomial function, that the graph is a smooth curve. Notice that when *x*=2, the *y*-value is negative, but when *x*=3 the *y*-value is positive. What do you think must happen somewhere between *x*=2 and *x*=3?

Now, remember that when we find the roots of a polynomial function, we are looking for the values of *x* where the graph crosses the *x*-axis. What statement can you make about one of the roots of this polynomial?

The Location Principle tells us that for a polynomial function, f(x), if f(a) is negative and f(b) is positive, then a zero or root exists between *x=a and x=b.*

Now, we know that this function, $f(x) = x^3 - 2x^2 + x - 3$, MUST have a real root between *x*=2 and *x*=3. Can we be more accurate than that? Fill in the table below:

x	**f(*x*)**
2.0	
2.2	
2.4	
2.6	
2.8	
3.0	

A real root for this function must be between ______________ and ________________.

How can we be more accurate? **Describe a process that would find the value of this zero to the nearest <u>tenth</u> and try it!** *To check your answer, use a graphing calculator to find the zero.*

The function in this example only had one real root, the other two are imaginary. If a function has more than one real root you would need to repeat this process for each real root!