Advanced Algebra
Polynomial Functions: Approximating the Real Zeros of a Polynomial

Sometimes, there are no *rational* zeros for a polynomial function. This doesn't mean that the graph never crosses zero or that there are no *real* solutions to the polynomial equation. When this is the case, the Rational Root Theorem and the process of dividing polynomials will not help.

Graph the polynomial function $y = 2x^3 + x^2 - 7x - 4$ on a graphing calculator and notice where the curve seems to cross the x-axis. Any place where the graph crosses the x-axis indicates a real root. Imaginary roots do not show up on the graph.

<u>Graphing Calculator Solution</u>
You can use your calculator to find the approximate values of the roots or *x*-intercepts. You can try the TRACE feature on your calculator, but you might get a more accurate result if you use the X-INT or ZERO function. On a Casio these functions are usually in the "G-SOLVE" window; on a TI these functions are usually in the "Calc" or "Math" window.

<u>No Graphing Calculator?</u>
It is worth our time to consider how we could do this if our calculator didn't have this feature built in. After all, someone had to write the programs for these functions, and someday soon someone else will write a faster, more efficient program - maybe you will be that person!

Think for a moment, if all your calculator could do was add, subtract, multiply, divide and raise numbers to powers, how would you find the values of the zeros of this function? For instance, one zero is between -2 and -1, but is it -1.9, -1.6, -1.4, ...?

............think............think............think............think............think............think............think

<u>The Location Principle</u>

> If $y = f(x)$ is a polynomial function, and a and b are two numbers such that $f(a)$ is negative and $f(b)$ is positive, then the polynomial function $y = f(x)$ has at least one real zero between $x=a$ and $x=b$.

Consider our polynomial function: $y = 2x^3 + x^2 - 7x - 4$

We know that f`(-2) is negative (look at the graph) and f (-1) is positive, so there must be a root between *x* = -2 and *x* = -1. How can we get a better approximation for this root? Evaluate the function at the following values of *x*:

x	f(*x*)
-2	-2
-1.5	2
-1	2

Notice that the value of f (-2) is negative and the value of f (-1.5) is positive, this tells us that there is a zero between *x* = -2 and *x* = -1.5. We need to be more accurate than this though, so we need to find another f(*x*) between *x* = -2 and *x* = -1.5. Let's try somewhere near the middle, *x* = -1.7:

x	f(*x*)
-2	-2
-1.7	0.96
-1.5	2

Now we can see that the zero is between *x* = -2 and *x* = -1.7, let's try *x* = -1.9:

x	f(*x*)
-2	-2
-1.9	-0.81
-1.7	0.96

Notice that our f(x) values are getting closer to zero, that is because we are getting closer to the *x*-value that is the function's root. Now we see that the zero is between -1.9 and -1.7, let's try *x* = -1.8:

x	f(*x*)
-1.9	-0.81
-1.8	0.18
-1.7	0.96

Now we can see that the zero is actually somewhere between -1.9 and -1.8, but we only need to know to the nearest tenth. So, which one is closer to the actual zero? If your instincts say -1.8 is closer, you are correct, but we need to check the *x*-value between -1.9 and -1.8 to be completely sure:

x	f(*x*)
-1.9	-0.81
-1.85	-0.29
-1.8	0.18

Here we see that our actual root is between -1.85 and -1.80 which, to the nearest tenth, is -1.8.

This polynomial function, $y = 2x^3 + x^2 - 7x - 4$, has 2 other roots, one between x = -1 and 0 and one between x = 1 and 2.

x	f(x)
-1	2
0	-4
1	-8
2	2

Use the same process to determine those values to the nearest tenth as well. Since you get to choose your initial values, and that choice determines the next choice, I cannot tell you how many steps it will take. You should be choosing numbers to a tenth, until you know between which tenths the actual solutions lie. Then perform a final calculation with a value of x to hundredths place.

Your final solutions should be -0.6 and 1.9.