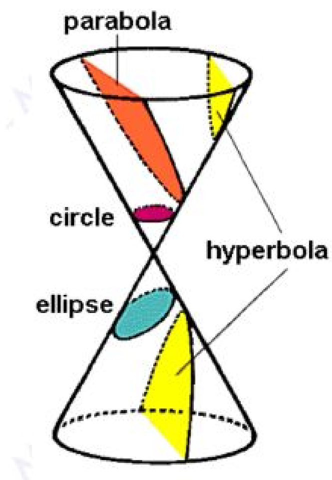
Advanced Algebra
Conics - Parabolas

Conics – General Information
Conics, or conic sections, are plane figures that are formed when you intersect a double-napped cone and a plane. The following diagram shows the different conics that can be formed by a double-napped cone being cut by a plane:



Parabolas: Notice that to create a parabola by intersecting a cone and a plane, the plane will pass through the base of the cone, but will only pass through one of the cones.

Any of these conics can be graphed on a coordinate plane. The graph of any conic on an *x*-*y* coordinate plane can be described by an equation in the form:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

This equation is the *General Form* for all conics. You will see as we study the different conics, their equations and graphs, that we can determine the type of conic given the equation based on the values of the coefficients, A, B, C, D, E and F.

Parabolas:
Looking at the General Form for conics, an equation in this form will graph a ***Parabola*** when:

- Either A or C, the coefficients of x^2 and y^2, must equal zero
- B, the coefficient of *xy*, must equal zero.
- D, E and F can equal any real values, D and E cannot both equal zero.

These equations would all form parabolas on the x-y coordinate plane:

$x^2 - 2x + 36y + 28 = 0$
$4y^2 - 50x - 75 = 0$
$9x^2 + 4y - 36 = 0$

To graph a ***Parabola*** given the equation in general form, we should first rewrite the equation into the ***Standard Form for Parabolas***:

$$y - k = a(x - h)^2 \quad \text{or} \quad x - h = a(y - k)^2$$

where (h, k) is the vertex of the parabola and $p = \frac{1}{4a}$ is the distance from the vertex to the *focus* and from the vertex to the *directrix*. If the *x*-term is squared, the parabola opens either up or down. If the *y*-term is squared the parabola opens to the right or left.

Converting an equation from General Form to Standard Form:

1) $4x^2 - 24x - 40y - 4 = 0$

$4x^2 - 24x = 40y + 4$

$4\left(x^2 - 6x\right) = 40y + 4$

$4\left(x^2 - 6x + \_?\_\right) = 40y + 4$

$\left(\frac{-6}{2}\right)^2 = 9$

$4\left(x^2 - 6x + \underline{9}\right) = 40y + 4 + \underline{4 \cdot 9}$

$4(x - 3)^2 = 40y + 40$

$\frac{4(x-3)^2}{40} = \frac{40y + 40}{40}$

$\frac{1}{10}(x - 3)^2 = y + 1$

$y + 1 = \frac{1}{10}(x - 3)^2$

1) Move constant & unpaired linear term to right.

2) Factor the coefficient of the squared term out of that variable group.

3) To Complete the Square, take half the linear coefficient and square it.

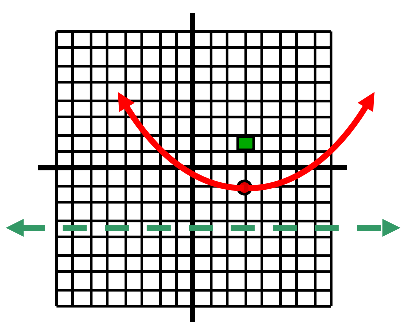
4) Add the squares to both sides, on right side multiply the square by the factor in front of the group.

5) Divide both sides by the coefficient of the unpaired linear variable.

From the Standard Form, determine the

Vertex: $(h, k) = (3, -1)$

Since '*x*' is squared, this parabola opens ***vertically***.
Since '*a*' is positive, it opens ***upward***.



Find the distance to the ***Focus***, p:

$$p = \frac{1}{4a} = \frac{1}{4 \cdot \frac{1}{10}} = \frac{10}{4} = 2.5$$

Focus: (3, 1.5)

The ***Directrix*** is a horizontal line below the parabola, 'p' units from the vertex.

Directrix: $y = -3.5$

The *Parabola* opens upward from the *Vertex*, away from the *Directrix*, around the *Focus*.

2) $y^2 + 2x = 0$

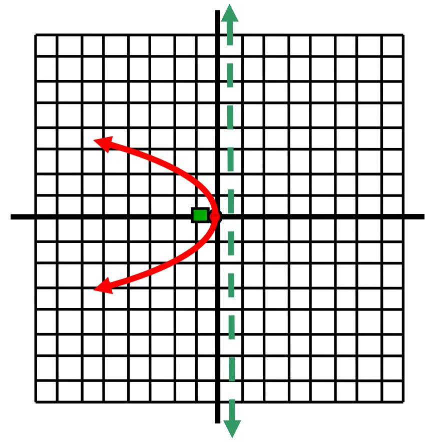
Since there is no linear y term, we do not complete the square.

$y^2 = -2x$

$\frac{y^2}{-2} = \frac{-2x}{-2}$

Divide both sides by the coefficient of the linear term.

$-\frac{1}{2}(y-0)^2 = (x-0)$



From the Standard Form, determine the

Vertex: (h, k) = (0, 0)

Since 'y' is squared, this parabola opens ***horizontally***.
Since 'a' is negative, it opens to the ***left***.

Find the distance to the ***Focus,*** *p:*

$$p = \frac{1}{4a} = \frac{1}{4 \cdot -\frac{1}{2}} = -\frac{1}{2}$$

Focus: (-0.5, 0)

The ***Directrix*** is a vertical line to the right of the parabola, 'p' units from the vertex.

Directrix: $x = 0.5$

The *Parabola* opens to the left from the *Vertex*, away from the *Directrix*, around the *Focus*.