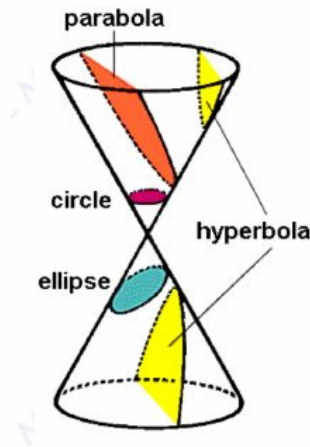


### Conics – General Information

Conics, or conic sections, are plane figures that are formed when you intersect a double-napped cone and a plane. The following diagrams show the different conics that can be formed by a double-napped cone being cut by a plane:



**Circles:** Notice that to create a circle by intersecting a cone and a plane, the plane must be parallel to the base of the cone.

Any of these conics can be graphed on a coordinate plane. The graph of any conic on an  $x$ - $y$  coordinate plane can be described by an equation in the form:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

This equation is the *General Form* for all conics. You will see as we study the different conics, their equations and graphs, that we can determine the type of conic given the equation based on the values of the coefficients, A, B, C, D, E and F.

### Circles:

Looking at the General Form for conics, for an equation in this form to graph a *Circle*,

- A and C, the coefficients of  $x^2$  and  $y^2$ , must be the same number and the same sign.
- B, the coefficient of  $xy$ , must equal zero.
- D, E and F can equal any real values.

*These equations would all form circles on the  $x$ - $y$  coordinate plane:*

$$x^2 + y^2 + 6x - 4y + 4 = 0$$

$$2x^2 + 2y^2 - 16y + 4 = 0$$

$$-3x^2 - 3y^2 + 9 = 0$$

To graph a circle given the equation in general form, we should first rewrite the equation into the *Standard Form for Circles*:

$$(x - h)^2 + (y - k)^2 = r^2$$

where (h, k) is the center of the circle and r = the radius.

**Converting a circle equation from General Form to Standard Form:**

Ex. 1)  $x^2 + y^2 + 6x - 4y + 4 = 0$   
 $x^2 + 6x + y^2 - 4y = -4$

1) Move constant to the right, rearrange terms to group like variables on left..

$$(x^2 + 6x + \underline{\quad}) + (y^2 - 4y + \underline{\quad}) = -4$$

$$\left(\frac{6}{2}\right)^2 = 9 \quad \left(\frac{-4}{2}\right)^2 = 4$$

2) Complete the Square for each variable set. Take half the linear coefficient & square it.

$$(x^2 + 6x + \underline{9}) + (y^2 - 4y + \underline{4}) = -4 + \underline{9} + \underline{4}$$

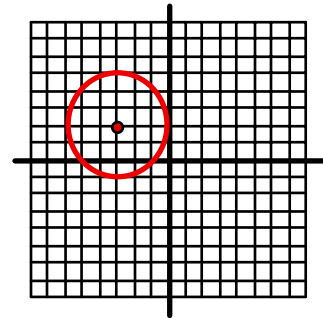
3) Add the squares to both sides.

$$(x + 3)^2 + (y - 2)^2 = 9$$

4) Now the trinomials are perfect squares, factor them.

From the *Standard Form*, determine the:

Center: (h, k) = (-3, 2)  
 Radius:  $r = \sqrt{9} = 3$

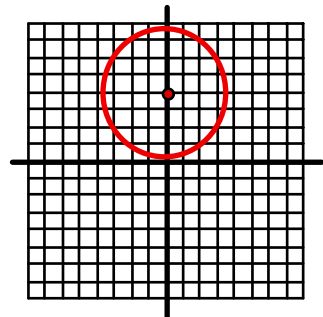


Ex. 2)  $2x^2 + 2y^2 - 16y + 4 = 0$   
 $x^2 + y^2 - 8y + 2 = 0$   
 $x^2 + y^2 - 8y = -2$   
 $x^2 + (y^2 - 8y + \underline{\quad}) = -2$   
 $x^2 + (y^2 - 8y + \underline{16}) = -2 + \underline{16}$   
 $(x - 0)^2 + (y - 4)^2 = 14$

\* Since A and C are not 1, first divide both sides by A.

From the *Standard Form*, determine the:

Center: (h, k) = (0, 4)  
 Radius:  $r = \sqrt{14} \approx 3.7$



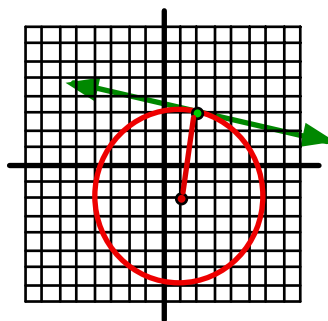
### Lines Tangent to a Circle:

Remember from Geometry, that if a line is tangent to a circle, it touches the circle once and is perpendicular to the radius of the circle at that point.

*Problem:* Find the equation of the line tangent to the circle  $(x - 1)^2 + (y + 2)^2 = 26$  at the point on the circle,  $(2, 3)$ .

Look at the graph:

The point  $(2, 3)$  is on the circle.  
A tangent line at  $(2, 3)$  has been drawn.



To find the equation of the tangent line, we must find the slope of the line. The tangent line is perpendicular to the radius, so first calculate the slope of the radius from the center  $(1, -2)$  to the point  $(2, 3)$ .

$$\frac{3 - (-2)}{2 - 1} = \frac{5}{1} = 5$$

Therefore, the slope of the *tangent* line (perpendicular to the radius) is  $-\frac{1}{5}$ .

Now the equation of the tangent line with the slope,  $-\frac{1}{5}$ , and the point,  $(2, 3)$ :

$$\begin{aligned}y - 3 &= -\frac{1}{5}(x - 2) \\y &= -\frac{1}{5}x + \frac{2}{5} + 3 \\y &= -\frac{1}{5}x + \frac{17}{5} \\x + 5y &= 17\end{aligned}$$